

Discussion on Event Horizon and Quantum Ergosphere of Evaporating Black Holes in a Tunnelling Framework

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In this paper, with the Parikh-Wilczek tunnelling framework the positions of the event horizon of the Vaidya black hole and the Vaidya-Bonner black hole are calculated respectively. We find that the event horizon and the apparent horizon of these two black holes correspond respectively to the two turning points of the Hawking radiation tunnelling barrier. That is, the quantum ergosphere coincides with the tunnelling barrier. Our calculation also implies that the Hawking radiation comes from the apparent horizon.

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I. INTRODUCTION

In 1974, Hawking published his astounding discovery that black holes radiate like a black body[1, 2]. Since then, black hole thermodynamics become a very interesting topic in theoretical physics. Since black hole radiates thermally, it must have entropy and other thermal properties. According to Hawking's original paper, the radiation comes from the vacuum fluctuation near the horizon. The idea is that when a virtual particle pair is created just outside the horizon, the negative energy virtual particle, which is forbidden outside, can tunnel inwards, and the positive energy virtual particle materializes as a real particle and escape to infinity. However, a black hole usually has event horizon and apparent horizon. Where does the Hawking radiation come from? At present, there are two different viewpoints about this problem. One is that the event horizon constitutes a black hole, and Hawking radiation comes from this surface or the neighborhood[3, 4]. In Ref.[5–9] an expression of local event horizon was obtained by assuming that the local event horizon is a null hypersurface and preserves the symmetries of the space time. Another viewpoint is that Hawking radiation comes from the apparent horizon, the laws of black hole thermodynamics and the expression of the Bekenstein-Hawking entropy are only suitable to this surface [10–16]. For a stationary black hole, the event horizon and the apparent horizon coincide with each other, these two different viewpoints do not cause problem. However, for a non-stationary black hole, the event horizon and the apparent horizon deviate from each other. Therefore, for non-stationary black holes it is very important for us to determine where the Hawking radiation comes from. In this paper we attempt to investigate this problem with Parikh-Wilczek tunneling framework and calculate the concrete positions of these two horizons in the Vaidya black holes and Vaidya-Bonner black holes. In General Relativity, the non-stationary black holes have their static counterparts, such as the Vaidya black holes correspond to the Schwarzschild black hole, and the Vaidya-Bonner black holes to the Reissner-Nordström black holes. In Parikh-Wilczek tunnelling

framework[17–19], the static background space time, such as the Schwarzschild black hole, is dynamical when self-gravitation of tunnelling particles is taken into account. Therefore, the treatment about the quantum tunnelling in Parikh-Wilczek’s tunneling framework describes the evolution of the non-stationary counterpart black hole. In this paper, by treating the tunnelling process of the Schwarzschild black hole as a volution of the Vaidya black hole, we first calculate the position of the event horizon of the Vaidya black holes, and then, we discuss the ergosphere and the location where Hawking radiation taking place. Similarly, the event horizon of the Vaidya-Bonner black holes is discussed too.

II. EVENT HORIZON AND QUANTUM ERGOSPHERE OF THE VAIDYA BLACK HOLES

Vaidya black hole is the simplest non-stationary black hole. Its line element is as follows

$$ds^2 = -(1 - \frac{2M(v)}{r})dv^2 + 2dvdr + r^2 d\Omega^2. \quad (1)$$

Ref.[5–9] give an equation about the local event horizon by suggesting that the event horizon is a null supersurface and preserves the symmetry of the space time, namely,

$$r_H = \frac{2M(v)}{1 - 2\dot{r}_H}, \quad (2)$$

where

$$\dot{r}_H = \frac{dr_H}{dv}. \quad (3)$$

The apparent horizon r_{AH} and the timelike limit surface r_{TLS} can be easily obtained respectively from the equations

$$\Theta|_{r=r_{AH}} = 0, \quad (4)$$

and

$$g_{vv} = 0, \quad (5)$$

where Θ is the expansion of the null rays. Namely,

$$r_{AH} = r_{TLS} = 2M(v). \quad (6)$$

Obviously, Eq. (2) is a to be solved equation about the event horizon. Generally, it is very difficult for us to obtain the exact expression of the event horizon. In the following we will try to obtain the concrete position by using the Parikh-Wilczek tunnelling framework.

In 2000, Parikh and Wilczek pointed out that Hawking’s previous calculation about the black hole radiation have a defect: energy conservation was not enforced during the emission process of a tunnelling particle[17–19]. If we consider the energy conservation, the particle’s self-gravitation should be taken into account. Then, the background space time should be dynamical, and therefore the spectrum of the Hawking radiation will not be thermal[17–54]. That is, when a particle (in order to preserve the symmetry of the space time, for the sake of simplicity, we take the tunneling particle as a sphere shell, a s-shell, as did in Ref.[17–19]) with mass ω tunnels out, the effect background space time will become a Schwarzschild black hole with the mass $M - \omega$. Therefore, the background space time is

in fact a evaporating Vaidya black hole space time. We can calculate the position of the event horizon of the Vaidya black hole by using the Parikh-Wilczek tunnelling framework.

let us first take into account a Schwarzschild black hole with the mass M . Its line element is as follows

$$ds^2 = -(1 - \frac{2M}{r})dt_s^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2d\Omega^2. \quad (7)$$

In the Parikh-Wilczek tunnelling framework, in order to calculate the emission rate of the tunnelling particles, Painlevé coordinates are adopted, that is, the line element of the Schwarzschild black hole space time should be written as the following form

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + 2\sqrt{\frac{2M}{r}}dtdr + dr^2 + r^2d\Omega^2. \quad (8)$$

However, to compare with the Vaidya black hole in literature, in this paper we adopt the ingoing Eddington-Finkelstein coordinates (v, r, θ, φ) by the transformation $v = t + r + \frac{1}{2M} \ln(\frac{r}{2M} - 1)$ from the Schwarzschild coordinate system $(t_s, r, \theta, \varphi)$ [55]. In this coordinate system, the line element reads

$$ds^2 = -(1 - \frac{2M}{r})dv^2 + 2dvdr + r^2d\Omega^2. \quad (9)$$

Obviously, if the mass of the black hole is a function of v , that is $M = M(v)$, then, the Eq.(9), the line element of the Schwarzschild black hole, will be the same as that of the Vaidya black hole. Similar to the Painlevé coordinate system, the ingoing Eddington-Finkelstein coordinate system have a series of good properties, and we can use it to describe the tunnelling process and calculate the emission rates of the tunnelling particles[55].

As mentioned above, in Hawking's original paper, Hawking radiation is described as a tunneling process triggered by vacuum fluctuations near the horizon. The idea is that when a virtual particle pair is created just outside the horizon, the negative energy virtual particle, which is forbidden outside, can tunnel inwards, and the positive energy virtual particle materializes as a real particle and escape to infinity. This process of tunneling can also be equivalently explained by Paikh and Wilczek as the image: there is a virtual particle pair created just inside the horizon, the positive energy virtual particle can tunnel out-no classical escape rout exists-where it materializes as a real particle. In either case, the negative energy particle is absorbed by the black hole, resulting in a decrease in the mass of the black hole, while the positive energy particle escape to infinity, appearing as Hawking radiation. According to Parikh-Wilczek tunneling framework, when a particle with a mass of ω tunnels out, the black hole will shrink from $r_H(M)$ to $r_H(M - \omega)$. It is the contraction of the black hole or the self-gravitation of the tunnelling particle set the barrier. In this model, the positions $r_H(M)$ and $r_H(M - \omega)$ correspond to two turning points. If we treat the dynamical Schwarzschild black hole as an evaporating Vaidya black hole, then, we can calculate the position of the event horizon.

According to the Parikh-Wilczek tunnelling framework, when a particle (a shell) tunnels out, the mass of the black hole decreases. Then, the event horizon will shrink. The tunnelling and the shrinking take place at the same time. Naturally, the tunnelling speed of a particle (a s-shell) is equal to that of the shrinking of the black hole. Therefore, we can obtain the shrinking velocity of the event horizon, namely,

$$\dot{r}_H = -\dot{r}, \quad (10)$$

where \dot{r} denote the velocity of the tunnelling particle (s-shell). In fact, for a s-shell, or a s wave, it is the velocity of the wave front.

There are two types of tunnelling particles. One is the massless particles, or null particles, which travel along a null geodesic. For this type of particles, the velocity of the wave front can be obtained by letting $ds^2 = d\theta = d\varphi = 0$, namely [17–19],

$$\dot{r} = \frac{1}{2}\left(1 - \frac{2M}{r}\right). \quad (11)$$

Another type of tunnelling particles are the non-zero-rest mass particles, or massive particles. Dislike the null particles, the tunnelling of the massive particles do not travel along the null geodesics. We can treat a massive particle as a de Broglie wave, and in order to preserve the spherical symmetry of the space time during the tunnelling process, we treat it as a spherical wave. Like the treatment in Ref.[20], the velocity of the wave front of the de Broglie s-wave is

$$\dot{r} = -\frac{1}{2}\frac{g_{00}}{g_{01}} = \frac{1}{2}\left(1 - \frac{2M}{r}\right). \quad (12)$$

Eqs. (11) and (12) show that the velocities of the wave fronts for the massless or massive particles are the same. Considering the self-gravitation, the mass M in Eqs. (9),(11),and (12) should be replaced with $M - \omega$. Therefore, we have

$$\dot{r}_H = -\dot{r} = -\frac{1}{2}\left(1 - \frac{2(M - \omega)}{r}\right). \quad (13)$$

In the following, we will calculate the concrete position of the event horizon of the Vaidya black hole by using Eq. (13). As described above, we take the dynamical Schwarzschild black hole as a Vaidya black hole. Before the particle tunnels out, the space time is a stationary Schwarzschild black hole space time, the event horizon, the apparent horizon, and the timelike limit surface locate at the same place, that is,

$$r_H = r_{AH} = r_{TLS} = 2M. \quad (14)$$

When particles tunnel out, the space time will become dynamic if self-gravitation is taken into account. To calculate the shrinking velocity of the event horizon at $r = r_{AH}$, we take $r = 2M$. Then, the shrinking velocity of the event horizon is

$$\dot{r}_H = -\dot{r}|_{2M} = -\frac{\omega}{2M}. \quad (15)$$

Therefore, the position of the event horizon of the Vaidya black holes is

$$r_H = \frac{2M(v)}{1 - 2\dot{r}_H} = \frac{2M}{1 + \frac{\omega}{M}} \approx 2(M - \omega). \quad (16)$$

Obviously, the event horizon r_H is a turning point of the tunnelling barrier. Since the apparent horizon and the timelike limit surface of the Vaidya black hole locate at the same place, and correspond to another turning point, namely,

$$r_{AH} = r_{TLS} = 2M, \quad (17)$$

we see that the ergosphere region coincide with the tunnelling barrier. That is, the ergosphere region corresponds to the classical inhibited region.

III. EVENT HORIZON AND QUANTUM ERGOSPHERE OF THE VAIDYA-BONNER BLACK HOLES

The line element of the Vaidya-Bonner black hole is

$$ds^2 = -(1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2})dv^2 + 2dvdr + r^2d\Omega^2. \quad (18)$$

Ref.[56] gives the expression of the local event horizon by suggesting that the event horizon is a null hypersurface and preserves the symmetry of the space time, namely,

$$r_H = \frac{M + \sqrt{M^2 - Q^2(1 - 2\dot{r}_H)}}{1 - 2\dot{r}_H}, \quad (19)$$

where

$$\dot{r}_H = \frac{dr_H}{dv}. \quad (20)$$

The apparent horizon and the timelike limit surface locate at

$$r_{AH} = r_{TLS} = M + \sqrt{M^2 - Q^2}. \quad (21)$$

Let us consider the corresponding stationary black hole, the Reissner-Nordström black hole. Its line element is

$$ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2}{r^2})dt^2 + (1 - \frac{2M}{r} + \frac{Q^2}{r^2})^{-1}dr^2 + r^2d\Omega^2. \quad (22)$$

There are two event horizons, and their expressions are

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2}. \quad (23)$$

We can easily get the surface gravity on the event horizon,

$$\kappa_{\pm} = \frac{r_+ - r_-}{2r_{\pm}^2}. \quad (24)$$

In order to compare with the Vaidya-Bonner black hole, we adopt the ingoing Eddington-Finkelstein coordinate system (v, r, θ, φ) . Let us do the transformation $v = t + r_*$, where

$$r_* = r + \frac{1}{2\kappa_+} \ln \frac{r - r_+}{r_+} - \frac{1}{2\kappa_-} \ln \frac{r - r_-}{r_-}. \quad (25)$$

Then, the line element of the Reissner-Nordström black hole can be written as

$$ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2}{r^2})dv^2 + 2dvdr + r^2d\Omega^2. \quad (26)$$

Comparing Eq. (26) with Eq. (18) we see that the Reissner-Nordström is a static counterpart of the Vaidya-Bonner black hole. Similarly, there are two types of tunnelling particles, the massless s-shell and the massive s-shell. The velocity of the wave front of the massless shell can be obtained by letting $ds^2 = d\theta = d\varphi = 0$, namely,

$$\dot{r} = \frac{1}{2}(1 - \frac{2M}{r} + \frac{Q^2}{r^2}). \quad (27)$$

For the massive particles, like the treatment in Ref.[20], the velocity of the wave front of the de Broglie s-wave is

$$\dot{r} = -\frac{1}{2} \frac{g_{00}}{g_{01}} = \frac{1}{2}(1 - \frac{2M}{r} + \frac{Q^2}{r^2}). \quad (28)$$

Eqs. (27) and (28) show that the velocities of the wave front for the massless or massive particles are the same. Considering the self-gravitation, the mass M in Eqs. (26),(27),and (28) should be replaced with $M - \omega$. Therefore, we have

$$\dot{r}_H = -\dot{r} = -\frac{1}{2}\left(1 - \frac{2(M - \omega)}{r} + \frac{Q^2}{r^2}\right). \quad (29)$$

Let $r = r_{AH} = M + \sqrt{M^2 - Q^2}$, then, the shrinking velocity of the event horizon can be written as

$$\dot{r}_H = -\dot{r}|_{r_{AH}} = -\frac{\omega}{M + \sqrt{M^2 - Q^2}}. \quad (30)$$

Substituting Eq. (30) into Eq. (19), we get the position of the event horizon of the Vaidya-Bonner black hole, namely,

$$r_H = \frac{M + \sqrt{M^2 - Q^2}(1 - 2\dot{r}_H)}{1 - 2\dot{r}_H} = (M - \omega) + \sqrt{M^2 - Q^2} - \frac{M^2}{\sqrt{M^2 - Q^2}} \frac{\omega}{M} + O\left(\frac{\omega}{M}\right)^2. \quad (31)$$

Since the tunnelling out point of the barrier in Parikh-Wilczek tunnelling framework is

$$r_f = (M - \omega) + \sqrt{(M - \omega)^2 - Q^2} = (M - \omega) + \sqrt{M^2 - Q^2} - \frac{M^2}{\sqrt{M^2 - Q^2}} \frac{\omega}{M} + O\left(\frac{\omega}{M}\right)^2, \quad (32)$$

we have

$$r_H = r_f = (M - \omega) + \sqrt{(M - \omega)^2 - Q^2}. \quad (33)$$

Similar to the dynamical Schwarzschild, the event horizon of the dynamical Reissner-Nordström black hole corresponds to the turning point $r_f = (M - \omega) + \sqrt{(M - \omega)^2 - Q^2}$, that is, the event horizon r_H is a turning point of the tunnelling barrier, and the ergosphere region coincide with the barrier. The ergosphere region corresponds to the classical inhibited region.

IV. WHERE DOES THE HAWKING RADIATION COME FROM?

Let us move to the final question: where does the Hawking radiation come from? According to the Parikh-Wilczek tunnelling framework, the ingoing tunnelling of a virtual negative energy particle is equivalent to the outgoing tunnelling of a real positive energy particle across the barrier from $r_i(M - \omega)$ to $r_f(M)$. Since the region between the event horizon r_H and the apparent horizon r_{AH} is the ergosphere, which is classically not able to detect, the emitted particle can only be detected outside the r_{AH} . In this meaning, we see that the Hawking radiation comes from the apparent horizon.

V. CONCLUSION

In above calculation, we treated the background space times in the Parikh-Wilczek tunnelling framework as a non-stationary counterparts. By using the Parikh-Wilczek tunnelling method the positions of the event horizons of the Vaidya black hole and the Vaidya-Bonner black holes were calculated. We find that the apparent horizon and the event horizon of these two types of black holes correspond to the two turning points of their tunnelling barriers. The barrier region corresponds to the ergosphere, and therefore the ergosphere is a classical inhibited region. Since the region between the event horizon and the apparent horizon is classically inhibited, we think that the Hawking radiation comes from the apparent horizon.

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